1. Introduction. In accordance with the representation constructed in [1, 2], the ordered rotation of the particles of a suspension under the influence of an external field leads to intensification of transport processes in the suspension. Since macroscopic motion of the suspension may be absent in this case, this intensification may appear to the observer to be the result of diffusion. In fact, the macroscopic effects are due to vortical flows around rotating particles. These flows act in a manner similar to a huge number of microscopic "agitators." Here, as regards diffusion phenomena, the suspension becomes anisotropic. The anisotropy is determined by the axial velocity vector of the internal rotation $\omega=\omega v$. Thus, the effective thermal conductivity [1]

$$
\begin{equation*}
\lambda_{i k}^{\prime}=\lambda_{i k} \lambda_{2}^{\theta}=\lambda_{0}^{\prime} \delta_{i k}+\lambda_{1}^{\prime} \varepsilon_{i h_{l}} v_{l}+\lambda_{2}^{\prime}\left(\delta_{i h}-v_{i} v_{b}\right) \tag{1.1}
\end{equation*}
$$

The coefficients of effective thermal conductivity $\lambda_{i}{ }^{\prime}$ in (1.1), measured in units of the thermal conductivity of the carrier fluid $\lambda_{2}{ }^{\circ}$, are found from dimensional analysis to be functions of the dimensionless parameters

$$
\varphi, \quad S=x_{1} / \kappa_{2}, \quad \Lambda=\lambda_{1}^{0} / \lambda_{2}^{0}, \quad \operatorname{Pr} c_{2}-\omega a^{2} / \mu_{2} .
$$

where $a, \varphi, \lambda_{1}{ }^{0}$, and $\kappa_{1}$ are the size, concentration, thermal conductivity, and diffusivity of the particles; $k_{2}$ is the diffusivity of the carrier fluid; $\mathrm{Pe}_{2}$ is the thermal Peclet number, characterizing the ratio of the rates of convective and conductive heat transfer in the fluid on the scale of the particles. Meanwhile, $\lambda_{0}^{\prime}=\lambda_{0}^{\prime}(\varphi, \Lambda), \lambda_{1}, 2^{\prime}=\lambda_{1}, 2^{\prime}(4, \Lambda, S$, $\mathrm{Pe}_{2}$ ).

Due to the analogy between the heat-transfer and diffusion equations of a neutral impurity, the result for the latter is obtained as a special case with $\lambda_{1}{ }^{0}=0$. Thus, regarding thermal conductivity, the relative values $\lambda_{1,2}{ }^{\prime}$ are numerically equal to the relative diffusion coefficient $D_{1,2}^{\prime}$. The diffusion coefficients were calculated by the cellular method in the $\operatorname{limit} \varphi, \mathrm{Pe}_{2} \ll 1$ in [2], where it was found that $\left(\mathrm{Pe}=\omega a^{2} / \mathrm{D}\right), \mathrm{D}_{1}^{1}=(1 / 2) \mathrm{m}^{1 / 3} \mathrm{Pe}$, $\mathrm{D}_{2}{ }^{1}=(3 / 4)_{\mathrm{P}^{2}}{ }^{3} \mathrm{Pe}^{2}$.

A similar calculation for thermal conductivity was performed in [3]. However, the difference from diffusion connected with the permeability of the particles was not manifest in the first approximation for $\mathrm{Pe}_{2}$ and $\Phi^{1 / 3}$ in this study. The calculation of thermal conductivity in [4] was also limited to small values of $\mathrm{Pe}_{2}$. Here, we obtain coefficients of effective thermal conductivity for a suspension which as a whole is at rest but in which internal rotation is taking place. The study is conducted for large concentrations and arbitrary parameters $\Lambda$ and $S$ in the range of Peclet numbers from 0 to $10^{3}$. The results are compared with experimental findings [5].
2. Tensor of Effective Thermal Conductivity. Let us state the premises of the variant of the cellular method we will be using. We will also prove the validity of Eq. (1.1), linking the thermal symmetry of the medium with the symmetry of transport in the neighborhood of a particle. In constructing cellular models of media with randomly-distributed occlusions, investigators frequently replace the random distribution by an ordered distribution [6] to permit the use of periodicity conditions on the external surface of a cell. We will employ a different approach. Considering that macroscopic effects are produced only by the regular component of the hydrodynamic and temperature field in the neighborhood of a particle and that a random particle distribution means that this component can be determined by the regular characteristics of the process (the rate of ordered rotation and the macroscopic temperature gradient $G=G g)$, we assign the temperature profile in the particle and the surrounding fluid by means of the relation ( $i=1,2$ )

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$$
\begin{equation*}
t_{i}=\left|U_{i}(r)-\Phi_{i}(r)\right|(\mathbf{g} \cdot \mathrm{r}) \quad \cdot \Phi_{i}(r) \mathrm{r} \cdot(\mathrm{~g} \times v)-F_{i}(r)(\mathrm{g} \cdot \boldsymbol{v})(\mathrm{r} \cdot \boldsymbol{v}) . \tag{2.1}
\end{equation*}
$$

The subscripts 1 and 2 pertain the particle and the fluid, respectively; $U, F$, and $\Phi$ are unknown functions of the distance $r$ from the center of the particle. It is considered that when the temperature gradient and angular velocity are colinear, rotation does not disturb the temperature. The latter is given by the expression $t_{i}=U_{i}(r)(g \cdot r)(i=1$, 2 ). Thus, in relation to the particle we have isolated for examination, the suspension can be thought of as a homogeneous isotropic medium. In accordance with this, a unit cell can be regarded as a sphere having the radius $R=\varphi^{-1 / 3} a$ and the volume $V_{0}$. This volume is equal to the volume of the suspension calculated per particle.

Let us change over to dimensionless quantities. To do this, we introduce the scale of distance $a$ (particle radius) and temperature $\Delta t=G a$. We assume that the observed heat flux is equal to the cell-averaged local value $q$. With allowance for $\nabla \cdot q=0$,

$$
\begin{equation*}
\left\langle q_{h}\right\rangle==V^{-1} \int_{V} q_{k} d V=V^{-1} \int_{V} \partial\left(x_{i}, \xi_{i}\right) / \Delta x_{i} d V . \tag{2.2}
\end{equation*}
$$

The unit of measurement of heat flux is $\lambda_{2}{ }^{\circ} \mathrm{G}$. In the particle and the fluid

$$
\begin{equation*}
\mathbf{q}_{1}=\left(c_{1} \cap_{1} \omega a^{2} / \lambda_{2}^{0}\right) \mathbf{v}_{1} t_{1}-\Lambda \nabla t_{1}, \mathrm{q}_{2}=\left(c_{2} \rho_{2} \omega a^{2} / \lambda_{2}^{n}\right) \mathbf{v}_{2} t_{2}-\nabla t_{2} \tag{2.3}
\end{equation*}
$$

( $c$ and $\rho$ are heat capacity and density; $v$ is velocity). Changing over in (2.2) to integration over the surface and considering that no fluid flow occurs either through the surface of the particle or through the surface $\Sigma$ and the volume $V-$ and also keeping in mind that the normal component of the heat flux is continuous - we obtain

$$
\begin{equation*}
\left\langle q_{k}\right\rangle=--V^{-1} \int_{\Sigma} x_{k}\left(\partial t_{2} / \partial x_{i}\right) n_{i} d \Sigma . \tag{2.4}
\end{equation*}
$$

Inserting temperature distribution (2.1) into (2.4) and considering that $\left\langle n_{i} n_{k}\right\rangle=\delta_{i k} / 3$, we find the cell-averaged heat flux in the form $\left\langle q_{i}\right\rangle=-\lambda_{i k}{ }^{\prime} g_{k}$, where the tensor $\lambda_{i k}{ }^{\prime}$ is determined from Eq. (1.1)

$$
\begin{gather*}
\lambda_{0}^{\prime}=U_{2}^{\prime}(R) R+  \tag{2.5}\\
U_{2}(R), \lambda_{1}^{\prime}=\Phi_{2}^{\prime}(R) R+\Phi_{\mathrm{q}}(R), \lambda_{2}^{\prime}=F_{2}^{\prime}(R) R+ \\
\\
+F_{2}(R) \quad\left(U^{\prime}=d U / d r \ldots\right) .
\end{gather*}
$$

The coefficients of effective thermal conductivity are expressed through the values of the functions $F, U$, and $\Phi$ and their derivatives on $\Sigma$.
3. Calculation of the Coefficients of Effective Thermal Conductivity. Equation (2.3) was obtained on the assumption that the observed temperature gradient is assigned. This imposes the condition $V^{-1} \int_{\Sigma} t_{2} \mathbf{n} d \Sigma=g$ on profile (2.1).

By virtue of the random distribution of particles in the volume, satisfying this condition, the regular part of the temperature on $\Sigma$

$$
\begin{equation*}
t_{2}(R)=\mathrm{g} \cdot \mathbf{R} \tag{3.1}
\end{equation*}
$$

The equation of heat transfer in the cell has the form

$$
\begin{equation*}
\mathbf{v}_{i} \cdot \nabla_{i}=\mathrm{Pe}_{i}^{-1} \nabla^{2} l_{i} \quad(i=1,2) . \tag{3.2}
\end{equation*}
$$

Here, velocity is measured in the units $\omega a ; \mathrm{Pe}_{\mathrm{i}}=\omega a^{2} / \kappa_{i}$. Since heat convection does not play a role in such a small volume, the velocity $v_{2}$ is assumed to be constant. For velocity, we adopt a Stokes profile between rotating and stationary spheres, since the velocity of the fluid is extinguished by the surrounding particles at the distance $\sim$ R:

$$
\begin{equation*}
\mathbf{v}_{i}=f_{i}(r) \boldsymbol{v} \times \mathbf{r}, f_{1}=1, f_{2}=\left(r^{-3}-\varphi\right) /(1-\varphi) . \tag{3.3}
\end{equation*}
$$

As is known [7], Eq. (3.3) is valid for $v_{2}$ only for small Reynolds numbers ( $\operatorname{Re}=\omega a^{2} / v_{2}$, $\nu_{2}$ is the kinematic viscosity of the fluid). Introducing the Prandtl number $\operatorname{Pr}=\nu_{2} / \kappa_{2}$, we write $\mathrm{Pe}_{2}=\operatorname{RePr}$. It follows from this that the restriction on the Reynolds number (Re < 1) leads to a restriction on the Peclet numbers ( $\mathrm{Pe}_{2}<\mathrm{Pr}$ ). For typical fluids $\left(v_{2}=10^{-5}\right.$. $10^{-6} \mathrm{~m}^{2} / \mathrm{sec}, k_{2}=10^{-7} \mathrm{~m}^{2} / \mathrm{sec}$ ), this number should not exceed $10-10^{2}$.

We obtain the equations for the unknown functions after inserting (2.1) and (3.3) into (3.2):

$$
\begin{equation*}
U_{i}^{\prime \prime}+(4 / r) U_{i}^{\prime}=0, F_{i}^{\prime \prime}+(4 / r) F_{i}^{\prime}=-\mathrm{Pe}_{i} f_{i}(r) \Phi_{i}, \Phi_{i}^{\prime \prime}+(4 / r) \Phi_{i}^{\prime}=\mathrm{Pe}_{i} f_{i}(r)\left(F_{i}+U_{i}\right) . \tag{3.4}
\end{equation*}
$$

Along with condition (3.1) on $\Sigma$, at the center and on the surface of the particle we have $t_{1}(0) \neq \infty, t_{1}(1)=t_{2}(1), t_{1}^{\prime}(1)=\Lambda^{-1} t_{2}^{\prime}(1)$. This leads to the following ( $\zeta=F, \Phi, U$ ):

$$
\begin{gather*}
\left.\zeta_{1} r\right|_{r=0} \neq \infty, U_{2}(R)=1, F_{2}(R)=\Phi_{2}(R)=0, \zeta_{1}(1)=\zeta_{2}(1), \\
\left(\zeta_{1}+r_{\zeta}^{\prime}\right)_{r=1}^{\prime}=\Lambda^{-1}\left(\zeta_{2}+r \zeta_{2}^{\prime}\right)_{r=1} . \tag{3.5}
\end{gather*}
$$

The first equation of (3.4) is solved separately:

$$
\begin{equation*}
U_{i}=C_{i 1}+C_{i 2} / r^{3}(i=1,2) \tag{3.6}
\end{equation*}
$$

Here, $C_{11}=3 /(\Lambda+2+\varphi(1-\Lambda)) ; C_{22}=(1-\Lambda) /(\Lambda+2+\varphi(1-\Lambda)) ; C_{21}=0 ; C_{12}=(\Lambda+$ $2) /(\Lambda+2+\varphi(1-\Lambda))$. The functions $F$ and $\Phi$ are sought in the form of power series. Inside the particle, satisfying the condition at zero, we obtain expansions that are accurate to within the two arbitrary constants $\alpha_{0}$ and $\alpha_{2}$ :

$$
\begin{equation*}
\Phi_{1}=\alpha_{0} \Phi_{10}+\alpha_{2} \Phi_{12}, F_{1}=-C_{11}+\alpha_{2} F_{10}-\alpha_{0} F_{12} \tag{3.7}
\end{equation*}
$$

where $\Phi_{10}=\sum_{h=1}^{\infty} a_{k} r^{1\left(\frac{1}{2}-1\right)} ; \Phi_{32}=\sum_{k=1}^{\infty} b_{k} r^{4 k-2} ;$

$$
F_{1 \theta}=10 / \mathrm{Pe}_{1}-\sum_{k=1}^{\infty}\left[\mathrm{Pe}_{1} b_{k} / \mathscr{P}_{2}(4 k-2)\right] r^{4 h} ; H_{12}=\sum_{k=1}^{\infty}\left[\mathrm{Pe}_{1} a_{k} / \mathscr{P}_{2}(4 k-4) r^{4 h-2},\right.
$$

while the following recursion formulas exist:

$$
\begin{gathered}
a_{1}=b_{1}=1, a_{k+1}=-\mathrm{Pe}_{1}^{3} a_{k} / \mathscr{P}_{4}(4 k-4), b_{k+1}=-\mathrm{Pe}_{1}^{2} b_{k} / \mathscr{P}_{4}(4 k-2), \\
\mathscr{P}_{2}(m)=(m+2)(m+5), \mathscr{P}_{4}(m)=\mathscr{P}_{2}(m) \mathscr{P}_{2}(m+2) .
\end{gathered}
$$

Outside the particle, we introduce the variable $x=R / r-1$. This allows us to avoid negative powers of $r$ and to satisfy the condition on $\Sigma$. We write

$$
\begin{equation*}
\Phi_{2}=\sum_{k=1}^{\infty} A_{k} x^{n}, \quad F_{2}=\sum_{h=1}^{\infty} B_{l} x^{l_{k}} \tag{3.8}
\end{equation*}
$$

After insertion of (3.8) into (3.6)

$$
\begin{gathered}
A_{2}=A_{1}, A_{3}=A_{1} \beta+D\left(C_{12}-C_{22} \varphi\right) / 2, A_{4}=\frac{1}{4} D\left(B_{1}+C_{22} \varphi-2 C_{12}\right), \\
A_{5}=D\left(3 C_{12}-B_{1}\right) / 5, A_{3}=\left(7_{1} 30\right)\left(B_{1}-3 C_{12}\right) D, A_{7}=(D / 20)\left(C_{12}-B_{1} / 3\right), \\
A_{5}=(27 D D 60)\left(3 C_{12}-B_{1}\right)-3 D^{2} A_{1} 1120, B_{2}=B_{1}, \\
B_{3}=B_{1} 3, B_{4}=-D A_{1} / 4, B_{5}=D A_{1} / 5, D=\operatorname{Pe}_{2} \varphi^{1 / 3} /(1-\varphi), \\
B_{k+2}=-[(k+1)(k+2)]^{-1}\left[M_{k}(B)+D L_{k}(A)\right](k \geqslant 4), \\
A_{h+2}=-[(k+1)(k+2)]^{-1}\left[M_{k}(A)-D L_{k}(B)\right](k \geqslant 7), \\
M_{k}(B)=(k-2)(k-5) B_{k-2}+2(k-1)(2 k-5) B_{k-1}+ \\
-6 k(k-2) B_{k}+2(k+1)(2 k-1) B_{k+1}, \\
L_{k}(A)=3 A_{k-1}+3 A_{k-2}+A_{k-3} .
\end{gathered}
$$

All $A_{k}$ and $B_{k}$ are determined through $A_{1}$ and $B_{1}$. Thus, introducting the notation $F_{2}=F_{2}\left(A_{1}\right.$, $B_{1}$ ), $\Phi_{2}=\Phi_{2}\left(A_{1}, B_{1}\right)$, we find $\Phi_{2}=\Phi_{20}+A_{1} \Phi_{21}+B_{1} \Phi_{22}, F_{2}=F_{20}+A_{1} F_{21}+B_{1} F_{22}$. Here (as well as for $F), \Phi_{20}=\Phi_{2}(0,0), \Phi_{21}=\Phi_{2}(1,0)-\Phi_{2}(0,0), \Phi_{22}=\Phi_{2}(0,1)-\Phi_{2}(0,0)$.

Consequently, the sought functions are determined to within four unknowns: $\alpha_{0}, \alpha_{2}$, $A_{1}$, and $B_{1}$. For these unknowns, we obtain the following algebraic system after we insert (3.7) and (3.4) into the boundary conditions for the particle surface (3.6)

$$
\begin{equation*}
A_{i k} y_{k}=D_{i}, \tag{3.9}
\end{equation*}
$$

where $\mathrm{y}=\left(\alpha_{0}, \alpha_{2}, \mathrm{~A}_{1}, \mathrm{~B}_{1}\right)$;

The first subscript of $A_{i k}$ is the row number. The values of the functions and their derivatives in A and D are taken on the particle surface.

In accordance with (2.6), (3.5), (3.6), and (3.8), the coefficients of thermal conductivity are calculated from the formulas

$$
\begin{gather*}
\lambda_{0}^{\prime}=1+3 \varphi(\lambda-1) \mu(1+2+\varphi(1-A)), \lambda_{2}^{\prime}=-\left.F_{2}^{\prime}\right|_{x=0}=-B_{1},  \tag{3.10}\\
\lambda_{2}^{\prime}=-\left.\Phi_{2}^{\prime}\right|_{x=0}=-A_{1} .
\end{gather*}
$$

The first of these formulas was determined in explicit form, while the others are found from the solution of system (3.9). The result for $\lambda_{0}{ }^{\prime}$ repeats the familiar Maxwell formula.
4. Analysis of the Results. Actual media in which transport occurs by internal rotation are magnetic suspensions and colloids (magnetic fluids). First of all, it should be noted that the condition $\mathrm{Pe}_{2} \simeq 1$ - which can be used to evaluate the possibility of observing the rotation effect - requires fairly coarse particles ( $\sim 10 \mu \mathrm{~m}$ ) when allowance is made for the usual values $\omega=10^{2}-10^{4} \mathrm{sec}^{-1}$ and the value of diffusivity $\kappa_{2}=10^{-7} \mathrm{~m}^{2} / \mathrm{sec}$ that is typical of fluids. Suspensions of this size contain individual particles, while magnetic fluids should contain the corresponding size of aggregate. Such formations have been observed repeatedly (see [8], for example). We will examine three specific objects: suspensions of iron particles, magnetite particles, and impermeable particles (diffusion) in an organic liquid (kerosene). We will also examine a colloid of magnetite in kerosene. The coefficients of thermal conductivity and diffusivities used for kerosene, magnetite, and iron were respectively equal to $\lambda=0.13,6.26$, and $126 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}) ; \mathrm{k}=0.76 \cdot 10^{-7}, 0.21 \cdot 10^{-5}$, and $0.47 \cdot 10^{-4} \mathrm{~m}^{2} / \mathrm{sec}$. For the carrier and magnetite, we also need $\rho=0.82 \cdot 10^{3}$ and $5 \cdot 10^{3}$ $\mathrm{kg} / \mathrm{m}^{3}$ and $\mathrm{c}=2.09$ and $0.57 \mathrm{~kJ} /(\mathrm{kg} \cdot \mathrm{K})$. In the graphs, lines $1-4$ corresond to particles of iron and magnetite, magnetite aggregates, and the impermeable particles.

Let us first discuss the behavior of the coefficient $\lambda_{0}{ }^{\prime}$, which with fixed properties of the components depends only on the concentration of the solid phase and is measured at a state of rest in a zero field. The relation $\lambda_{0}{ }^{\prime}(\varphi)$ is shown in Fig. 1. It should be recalled that the values of $\lambda_{0}{ }^{\prime}$ are measured in units of the thermal conductivity of the carrier.

Returning to experimental data on the concentration dependence of the thermal conductivity of magnetite magnetic fluids [9], we find the empirical formula $\lambda_{0}{ }^{\prime}=1+k \varphi_{m}$ with $k=4.5$ for the initial section of this relation. In accordance with Eq. (3.12), we have $\lambda_{0}^{\prime}=1+k^{\prime}\left(f, k^{\prime}=3(\Lambda-1) /(\Lambda+2)\right.$ on the linear section. Meanwhile $(\Lambda=48) k^{\prime}=2.82$ for magnetite in kerosene. The difference between $k$ ' and $k$ can be attributed to the difference in the concentration of the solid phase $\varphi$ from the magnetic concentration $\varphi_{m}$ used in the empirical relation in [9]. This difference, in turn, exists because of the presence on the particles of a nonmagnetic surface layer [10] with a thickness on the order of the lattice constant $\delta=8 \cdot 10^{-4} \mu \mathrm{~m}$. This layer forms with the chemical deposition of a surfaceactive substance (surfactant). Taking the mean radius of the magnetic core of the particle $R$ to be equal to $5 \cdot 10^{-3} \mu \mathrm{~m}$, we find $\varphi=\varphi_{m}(1+\delta / R)^{3}=1.56 \varphi_{m} . \quad$ From this, $k=k^{\prime} \varphi / \varphi_{m}=4.4$, which agrees with the experimental data to within $2 \%$. The points in Fig. 1 show the data in [9] for magnetite in kerosene recalculated with the use of the above relation between $\varphi$ and $\varphi_{m}$.

We begin our examination of transport by internal rotation with an analysis of an experi ment involving intensification of heat transfer through a magnetic fluid rotating about the axis of a cylindrical layer under the influence of a transverse magnetic field [5]. In this


Fig. 1


Fig. 2


Fig. 4
geometry, $g \cdot v=0$, and the measured increment of the coefficient of effective thermal conductivity is numerically equal to $\lambda_{2}$ '. Since (as noted above) a marked increase in heat transfer in a magnetic fluid due to rotation is possible only in the presence of aggregates, it is necessary to evaluate the thermophysical properties of the latter. The magnetic concentra tion of particles in the test specimen was 0.06 , so the concentration of the solid phase was 0.094 . The fraction of the volume occupied by the aggregates was equal to the hydrodynamic concentration - including the volume occupied by surfactant layers. The hydrodynamic concentration is usually evaluated from the increment of the effective viscosity of the magnetic fluid in the field. However, such estimates yield very conflicting results, since this increment also depends to a significant extent on the form of the particles (aggregates) - which is usually unknown. We will determine it from the formula $\varphi_{h}=\varphi\left(1+\delta_{1} / R_{1}\right)^{3}$, where $\delta_{1}$ is the length of a surfactant molecule. It is equal to $2 \cdot 10^{-3} \mu \mathrm{~m}$ for oleic acid. The quantity $R_{1}$ in the formula is the radius of the particle. Taking $R_{1}=5 \cdot 10^{-3} \mu \mathrm{~m}$ and $\varphi=$ 0.094 , we obtain $\varphi_{h}=0.25$. In the present case, the volume fraction of particles in aggregates $\varphi^{\prime}=\varphi / \varphi_{h}=0.38$. Using this value, we find from Eq. (3.10) for the thermal conductivity of the aggregate $\lambda_{1}{ }^{0}=2.7 \lambda_{2}{ }^{0}\left(\lambda_{2}{ }^{0}\right.$ is the thermal conductivity of kerosene), i.e., $\Lambda=2.7$. Calculating the heat capacity of a unit volume of aggregate from the formula $c^{\prime}=$ $\rho_{b} \varphi^{\prime} c_{b}+\rho_{f}\left(1-\varphi^{\prime}\right) c_{f}$, we also obtain an estimate for its diffusivity $\kappa_{1}=\lambda_{1}{ }^{0} / c^{\prime}=1.6 \cdot 10^{-7}$ $\mathrm{m}^{2} / \mathrm{sec}$. Then $\mathrm{S}=\mathrm{k}_{1} / \mathrm{K}_{2}=2.1$. Figure 2 shows coefficients $\lambda_{1}^{\prime}$ and $\lambda_{2}^{\prime}$ calculated with $\Lambda=2.7, S=2.1$, and $\varphi=0.25$ and referred to the thermal conductivity of the colloid $\lambda_{0}{ }^{\prime}=$ 1.29. For comparison, we take the data in [5] in a Maxwell field ( $H=1500$ Oe), when the
aggregates are least subject to hydrodynamic disintegration and are nearly completely stopped by the field. Here they rotate inside the fluid with the empirically measured rate of rotation of the layer. We have a single undetermined parameter - the size of an aggregate $a$ to establish agreement between theory and experiment. Agreement is obtained with a choice of $a=110 \mu \mathrm{~m}$. Data from [5] recalculated in accordance with this value is shown by the points in Fig. 2. The agreement between theory and experiment is very good. We should point out the qualitative aspect of the relation: $\lambda_{2}{ }^{\prime}$ increases quadratically on the initial section and then slows, with the increase becoming close to linear. As regards the quantitative value of $x$, it may be overstated due to the nonspherical form of aggregates stretched out by the field. In accordance with Fig. 2, the increase in $\lambda_{1}{ }^{\prime}$ with $P e_{2}$ is at first linear and then also slows.

These relations are shown in Fig. 3a, b in a logarithmic scale within a broader range of $\mathrm{Pe}_{2}$ for $\underline{\varphi}=0.25$. The calculations were performed up to $\mathrm{Pe}_{2} \simeq 10^{3}$. At large values of $\mathrm{Pe}_{2}$, the series for the functions $\Phi_{2}$ and $\mathrm{F}_{2}$ diverge. It should be recalled that such large values of $\mathrm{Pe}_{2}$ exceed the earlier-indicated boundary connected with the use of the Stokes profile for flow in a microscopic eddy. The coefficients $\lambda_{1}{ }^{\prime} / \lambda_{0}{ }^{\prime}$ and $\lambda_{2}{ }^{\prime} / \lambda_{0}$ are greater for particles with a lower thermal conductivity. Meanwhile, we have an approximate equality (in order of magnitude) $\lambda_{1}{ }^{2}{ }^{2} \approx \lambda_{2}^{\prime}$. It is interesting that the functions $\lambda_{1}{ }^{\prime}\left(\mathrm{Pe}_{2}\right)$ and $\lambda_{2}{ }^{\prime}\left(\mathrm{Pe}_{2}\right)$ become nonmonotonic for large values of $\mathrm{Pe}_{2}$. This is difficult to explain, but such behavior persists with an increase in the number of terms in the series and doubling of the accuracy of the entire computation. The problem of the divergence of the series for $\mathrm{F}_{2}$ and $\Phi_{2}$ also arises for low concentrations. As a result, the concentration dependence of the coefficients $\lambda_{1}^{\prime}$ and $\lambda_{2}^{\prime}$ was calculated only for $\varphi \geq 0.2$. The results are shown in Fig. 4a, b (curves 1,3 , and 4 for $\mathrm{Pe}_{2}=100,15$, and 10 ). The calculations were performed up to $\varphi=1$. However, they lose meaning (as indicated by the dotted lines) as a dense packing is approached. The dashed lines show the results of extrapolation to the region of low concentrations. It can be seen that the character of the relations is to a large extent determined by the relationship between the thermophysical properties of the fluid and particles. Thus, the relation $\lambda_{1}{ }^{\prime} / \lambda_{0}{ }^{\prime}=f(\varphi)$, increasing monotonically with $\varphi$ for the impermeable particles, acquires a gentler slope for a suspension with a fluid and particles having roughly the same thermophysical characteristics (aggregated magnetic fluid). The relation exhibits a fairly sharp maximum in the neighborhood $q \simeq 0.25$, when the thermal conductivity of the particles is considerably greater than the thermal conductivity of the carrier fluid. The relation $\lambda_{2}{ }^{\prime} / \lambda_{0}{ }^{\prime}=\mathrm{f}(\varphi)$ is monotonic in character when the thermophysical properties of the particles and fluid are the same; the maximum appears when the thermal conductivity of the particles deviates from that of the fluid in either direction.

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