HEAT AND MASS TRANSFER BY INTERNAL ROTATION WITH ARBITRARY PECLET NUMBERS

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1. Introduction. In accordance with the representation constructed in [1, 2], the ordered rotation of the particles of a suspension under the influence of an external field leads to intensification of transport processes in the suspension. Since macroscopic motion of the suspension may be absent in this case, this intensification may appear to the observer to be the result of diffusion. In fact, the macroscopic effects are due to vortical flows around rotating particles. These flows act in a manner similar to a huge number of microscopic "agitators." Here, as regards diffusion phenomena, the suspension becomes anisotropic. The anisotropy is determined by the axial velocity vector of the internal rotation $\omega = \omega v$. Thus, the effective thermal conductivity [1]

$$\lambda'_{ik} = \lambda_{ik}/\lambda_2^0 = \lambda_0^0 \delta_{ik} + \lambda_1^\prime \varepsilon_{ikl} v_l + \lambda_2^\prime (\delta_{ik} - v_i v_k).$$
(1.1)

The coefficients of effective thermal conductivity λ_i' in (1.1), measured in units of the thermal conductivity of the carrier fluid λ_2^0 , are found from dimensional analysis to be functions of the dimensionless parameters

$$\varphi$$
, $S := \varkappa_1 / \varkappa_2$, $\Lambda := \lambda_1^0 / \lambda_2^0$, $\operatorname{Pe}_2 = \omega a^2 / \varkappa_2$

where a, φ , λ_1^{0} , and κ_1 are the size, concentration, thermal conductivity, and diffusivity of the particles; κ_2 is the diffusivity of the carrier fluid; Pe₂ is the thermal Peclet number, characterizing the ratio of the rates of convective and conductive heat transfer in the fluid on the scale of the particles. Meanwhile, $\lambda_0' = \lambda_0'(\varphi, \Lambda)$, $\lambda_{1,2}' = \lambda_{1,2}'(\varphi, \Lambda, S, Pe_2)$.

Due to the analogy between the heat-transfer and diffusion equations of a neutral impurity, the result for the latter is obtained as a special case with $\lambda_1^0 = 0$. Thus, regarding thermal conductivity, the relative values $\lambda_{1,2}$ ' are numerically equal to the relative diffusion coefficient $D_{1,2}$ '. The diffusion coefficients were calculated by the cellular method in the limit φ , Pe₂ \ll 1 in [2], where it was found that (Pe = $\omega \alpha^2/D$), D_1 ' = (1/2) $\varphi^{1/3}$ Pe, D_2 ' = (3/4) $\varphi^{2/3}$ Pe².

A similar calculation for thermal conductivity was performed in [3]. However, the difference from diffusion connected with the permeability of the particles was not manifest in the first approximation for Pe_2 and $\varphi^{1/3}$ in this study. The calculation of thermal conductivity in [4] was also limited to small values of Pe_2 . Here, we obtain coefficients of effective thermal conductivity for a suspension which as a whole is at rest but in which internal rotation is taking place. The study is conducted for large concentrations and arbitrary parameters Λ and S in the range of Peclet numbers from 0 to 10^3 . The results are compared with experimental findings [5].

2. Tensor of Effective Thermal Conductivity. Let us state the premises of the variant of the cellular method we will be using. We will also prove the validity of Eq. (1.1), linking the thermal symmetry of the medium with the symmetry of transport in the neighborhood of a particle. In constructing cellular models of media with randomly-distributed occlusions, investigators frequently replace the random distribution by an ordered distribution [6] to permit the use of periodicity conditions on the external surface of a cell. We will employ a different approach. Considering that macroscopic effects are produced only by the regular component of the hydrodynamic and temperature field in the neighborhood of a particle and that a random particle distribution means that this component can be determined by the regular characteristics of the process (the rate of ordered rotation ω and the macroscopic temperature gradient G = Gg), we assign the temperature profile in the particle and the surrounding fluid by means of the relation (i = 1, 2)

Minsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 124-130, July-August, 1989. Original article submitted December 16, 1987; revision submitted February, 15, 1988.

UDC 536.2

FRARY PECLET NUMBERS

$$t_i \coloneqq [U_i(r) \vdash \Phi_i(r)](\mathbf{g} \cdot \mathbf{r}) \vdash \Phi_i(r)\mathbf{r} \cdot (\mathbf{g} \times \mathbf{v}) - F_i(r)(\mathbf{g} \cdot \mathbf{v})(\mathbf{r} \cdot \mathbf{v}).$$
(2.1)

The subscripts 1 and 2 pertain the particle and the fluid, respectively; U, F, and Φ are unknown functions of the distance r from the center of the particle. It is considered that when the temperature gradient and angular velocity are colinear, rotation does not disturb the temperature. The latter is given by the expression $t_i = U_i(r)(g \cdot r)$ (i = 1, 2). Thus, in relation to the particle we have isolated for examination, the suspension can be thought of as a homogeneous isotropic medium. In accordance with this, a unit cell can be regarded as a sphere having the radius $R = \varphi^{-1/3} a$ and the volume V_0 . This volume is equal to the volume of the suspension calculated per particle.

Let us change over to dimensionless quantities. To do this, we introduce the scale of distance a (particle radius) and temperature $\Delta t = Ga$. We assume that the observed heat flux is equal to the cell-averaged local value q. With allowance for $\nabla \cdot q = 0$,

$$\langle q_h \rangle = V^{-1} \int_V q_h \, dV = V^{-1} \int_V \partial \langle x_h q_i \rangle / \partial x_i \, dV.$$
(2.2)

The unit of measurement of heat flux is λ_2^{0} G. In the particle and the fluid

$$\mathbf{q}_1 = \left(c_1 \rho_1 \omega a^2 / \lambda_2^0\right) \mathbf{v}_1 t_1 - \Lambda \nabla t_1, \ \mathbf{q}_2 = \left(c_2 \rho_2 \omega a^2 / \lambda_2^0\right) \mathbf{v}_2 t_2 - \nabla t_2$$
(2.3)

(c and ρ are heat capacity and density; v is velocity). Changing over in (2.2) to integration over the surface and considering that no fluid flow occurs either through the surface of the particle or through the surface Σ and the volume V - and also keeping in mind that the normal component of the heat flux is continuous - we obtain

$$\langle q_h \rangle = -V^{-1} \int_{\Sigma} x_h \left(\partial t_2 / \partial x_i \right) n_i d\Sigma.$$
 (2.4)

Inserting temperature distribution (2.1) into (2.4) and considering that $\langle n_i n_k \rangle = \delta_{ik}/3$, we find the cell-averaged heat flux in the form $\langle q_i \rangle = -\lambda_{ik}' g_k$, where the tensor λ_{ik}' is determined from Eq. (1.1)

$$\lambda_0 = U_2(R) R + U_2(R), \ \lambda_1' = \Phi_2'(R) R + \Phi_2(R), \ \lambda_2' = F_2'(R) R + F_2(R) - (U' = dU/dr \dots).$$
(2.5)

The coefficients of effective thermal conductivity are expressed through the values of the functions F, U, and Φ and their derivatives on Σ .

<u>3. Calculation of the Coefficients of Effective Thermal Conductivity.</u> Equation (2.3) was obtained on the assumption that the observed temperature gradient is assigned. This

imposes the condition $V^{-1}\int_{\Sigma} t_2 n \, d\Sigma = g$ on profile (2.1).

By virtue of the random distribution of particles in the volume, satisfying this condition, the regular part of the temperature on Σ

$$t_2(R) = \mathbf{g} \cdot \mathbf{R}. \tag{3.1}$$

The equation of heat transfer in the cell has the form

$$\mathbf{v}_i \cdot \nabla t_i = \mathrm{Pe}_i^{-1} \nabla^2 t_i \quad (i = 1, 2).$$
(3.2)

Here, velocity is measured in the units ωa ; $Pe_i = \omega a^2/\kappa_i$. Since heat convection does not play a role in such a small volume, the velocity v_2 is assumed to be constant. For velocity, we adopt a Stokes profile between rotating and stationary spheres, since the velocity of the fluid is extinguished by the surrounding particles at the distance -R:

$$\mathbf{v}_i = f_i(r)\mathbf{v} \times \mathbf{r}, f_1 = 1, f_2 = (r^{-3} - \varphi)/(1 - \varphi).$$
 (3.3)

As is known [7], Eq. (3.3) is valid for v_2 only for small Reynolds numbers (Re = $\omega a^2/v_2$, v_2 is the kinematic viscosity of the fluid). Introducing the Prandtl number $Pr = v_2/\kappa_2$, we write $Pe_2 = \text{RePr}$. It follows from this that the restriction on the Reynolds number (Re < 1) leads to a restriction on the Peclet numbers ($Pe_2 < Pr$). For typical fluids ($v_2 = 10^{-5}-10^{-6} \text{ m}^2/\text{sec}$, $\kappa_2 = 10^{-7} \text{ m}^2/\text{sec}$), this number should not exceed 10-10².

We obtain the equations for the unknown functions after inserting (2.1) and (3.3) into (3.2):

$$U_{i}^{'} + (4/r) U_{i}^{'} = 0, \ F_{i}^{''} + (4/r) F_{i}^{'} = -\operatorname{Pe}_{i}f_{i}(r) \Phi_{i}, \ \Phi_{i}^{''} + (4/r) \Phi_{i}^{'} = \operatorname{Pe}_{i}f_{i}(r) (F_{i} + U_{i}).$$
(3.4)

Along with condition (3.1) on Σ , at the center and on the surface of the particle we have $t_1(0) \neq \infty$, $t_1(1) = t_2(1)$, $t_1'(1) = \Lambda^{-1}t_2'(1)$. This leads to the following ($\zeta = F$, ϕ , U):

$$\begin{aligned} \zeta_1 r |_{r=0} &\neq \infty, \ U_2(R) = 1, \ F_2(R) = \Phi_2(R) = 0, \ \zeta_1(1) = \zeta_2(1), \\ (\zeta_1 + r\zeta_1')_{r=1} = \Lambda^{-1} (\zeta_2 + r\zeta_2')_{r=1}. \end{aligned}$$
(3.5)

The first equation of (3.4) is solved separately:

$$U_i = C_{i1} + C_{i2}/r^3 \ (i = 1, 2). \tag{3.6}$$

Here, $C_{11} = 3/(\Lambda + 2 + \varphi (1 - \Lambda))$; $C_{22} = (1 - \Lambda)/(\Lambda + 2 + \varphi (1 - \Lambda))$; $C_{21} = 0$; $C_{12} = (\Lambda + 2)/(\Lambda + 2 + \varphi (1 - \Lambda))$. The functions F and φ are sought in the form of power series. Inside the particle, satisfying the condition at zero, we obtain expansions that are accurate to within the two arbitrary constants α_0 and α_2 :

$$\Phi_{1} = \alpha_{0} \Phi_{10} + \alpha_{2} \Phi_{12}, F_{1} = -C_{11} + \alpha_{2} F_{10} - \alpha_{0} F_{12},$$
where $\Phi_{10} = \sum_{k=1}^{\infty} a_{k} r^{4(k-1)}; \quad \Phi_{12} = \sum_{k=1}^{\infty} b_{k} r^{4k-2};$
(3.7)

$$F_{10} = 10/\text{Pe}_1 - \sum_{k=1}^{\infty} [\text{Pe}_1 b_k / \mathcal{P}_2 (4k-2)] r^{4h}; \ F_{12} = \sum_{k=1}^{\infty} [\text{Pe}_1 a_k / \mathcal{P}_2 (4k-4)] r^{4h-2},$$

while the following recursion formulas exist:

$$a_{1} = b_{1} = 1, \ a_{k+1} = -\operatorname{Pe}_{1}^{2}a_{k}/\mathscr{P}_{4}(4k-4), \ b_{k+1} = -\operatorname{Pe}_{1}^{2}b_{k}/\mathscr{P}_{4}(4k-2), \mathscr{P}_{2}(m) = (m+2)(m+5), \ \mathscr{P}_{4}(m) = \mathscr{P}_{2}(m)\mathscr{P}_{2}(m+2).$$

Outside the particle, we introduce the variable x = R/r - 1. This allows us to avoid negative powers of r and to satisfy the condition on Σ . We write

$$\Phi_2 = \sum_{k=1}^{\infty} A_k x^k, \ F_2 = \sum_{k=1}^{\infty} B_k x^k.$$
(3.8)

After insertion of (3.8) into (3.6)

$$\begin{split} A_2 &= A_1, \ A_3 = A_1 \beta + D \left(C_{12} + C_{22} \varphi \right) / 2, \ A_4 &= \frac{1}{4} D \left(B_1 + C_{22} \varphi - 2 C_{12} \right), \\ A_5 &= D \left(3 C_{12} - B_1 \right) / 5, \ A_3 &= (7/30) \left(B_1 - 3 C_{12} \right) D, \ A_7 &= (D/20) \left(C_{12} - B_1 \right) 3 \right), \\ A_8 &= (247D/560) (3 C_{12} - B_1) - 3D^2 A_1 / 1120, \ B_2 &= B_1, \\ B_3 &= B_1 / 3, \ B_4 &= -DA_1 / 4, \ B_5 &= DA_4 / 5, \ D &= \text{Pe}_2 \varphi^{1/3} / (1 - \varphi), \\ B_{h+2} &= -\left[(k + 1)(k + 2) \right]^{-1} [M_h (B) + DL_h (A)] \ (k \geq 4), \\ A_{h+2} &= -\left[(k + 1)(k + 2) \right]^{-1} [M_h (A) - DL_h (B)] \ (k \geq 7), \\ M_h (B) &= (k - 2)(k - 5)B_{k-2} + 2(k - 1)(2k - 5)B_{h-1} + \\ &+ 6k \left((k - 2)B_k + 2 \left((k + 1)(2k - 1)B_{k+1} \right), \\ L_h (A) &= 3A_{h-1} + 3A_{h-2} + A_{h-3}. \end{split}$$

All A_k and B_k are determined through A_1 and B_1 . Thus, introducting the notation $F_2 = F_2(A_1, B_1)$, $\Phi_2 = \Phi_2(A_1, B_1)$, we find $\Phi_2 = \Phi_{20} + A_1\Phi_{21} + B_1\Phi_{22}$, $F_2 = F_{20} + A_1F_{21} + B_1F_{22}$. Here (as well as for F), $\Phi_{20} = \Phi_2(0, 0)$, $\Phi_{21} = \Phi_2(1, 0) - \Phi_2(0, 0)$, $\Phi_{22} = \Phi_2(0, 1) - \Phi_2(0, 0)$.

Consequently, the sought functions are determined to within four unknowns: α_0 , α_2 , A_1 , and B_1 . For these unknowns, we obtain the following algebraic system after we insert (3.7) and (3.4) into the boundary conditions for the particle surface (3.6)

$$A_{ik}y_k = D_i, (3.9)$$

where $y = (\alpha_0, \alpha_2, A_1, B_1);$

$$D = (\Phi_{20}, F_{20} + C_{11}, (\Phi_{20} - R\Phi'_{20})/\Lambda, C_{11} + (F_{20} - RF'_{20})/\Lambda);$$

$$A = \begin{vmatrix} \Phi_{10} & \Phi_{12} & -\Phi_{21} & -\Phi_{22} \\ -F_{12} & F_{10} & -F_{21} & -F_{22} \\ \Phi_{10} - \Phi'_{10} & \Phi_{12} - \Phi'_{12} & -(\Phi_{21} - R\Phi'_{21})\Lambda^{-1} & (\Phi_{22} - R\Phi'_{22})\Lambda^{-1} \\ -F_{12} - F'_{12} & F_{10} + F'_{10} & -(F_{21} - RF'_{21})\Lambda^{-1} & (F_{22} - RF'_{22})\Lambda^{-1} \end{vmatrix}$$

The first subscript of A_{ik} is the row number. The values of the functions and their derivatives in A and D are taken on the particle surface.

In accordance with (2.6), (3.5), (3.6), and (3.8), the coefficients of thermal conductivity are calculated from the formulas

$$\lambda'_{0} = 1 + 3\varphi (\Lambda - 1)/(\Lambda + 2 + \varphi (1 - \Lambda)), \ \lambda'_{2} = -F'_{2}|_{x=0} = -B_{1}, \qquad (3.10)$$
$$\lambda'_{2} = -\Phi'_{2}|_{x=0} = -A_{1}.$$

The first of these formulas was determined in explicit form, while the others are found from the solution of system (3.9). The result for λ_0 ' repeats the familiar Maxwell formula.

4. Analysis of the Results. Actual media in which transport occurs by internal rotation are magnetic suspensions and colloids (magnetic fluids). First of all, it should be noted that the condition $Pe_2 \approx 1 - which can be used to evaluate the possibility of observing the rotation effect - requires fairly coarse particles (~10 µm) when allowance is made for the usual values <math>\omega = 10^{2}-10^{4} \text{ sec}^{-1}$ and the value of diffusivity $\kappa_2 = 10^{-7} \text{ m}^2/\text{sec}$ that is typical of fluids. Suspensions of this size contain individual particles, while magnetic fluids should contain the corresponding size of aggregate. Such formations have been observed repeatedly (see [8], for example). We will examine three specific objects: suspensions of iron particles, magnetite particles, and impermeable particles (diffusion) in an organic liquid (kerosene). We will also examine a colloid of magnetite in kerosene. The coefficients of thermal conductivity and diffusivities used for kerosene, magnetite, and iron were respectively equal to $\lambda = 0.13$, 6.26, and 126 W/(m·K); $\kappa = 0.76 \cdot 10^{-7}$, 0.21·10⁻⁵, and 0.47·10⁻⁴ m²/sec. For the carrier and magnetite, we also need $\rho = 0.82 \cdot 10^{3}$ and 5·10³ kg/m³ and c = 2.09 and 0.57 kJ/(kg·K). In the graphs, lines 1-4 corresond to particles of iron and magnetite, magnetite aggregates, and the impermeable particles.

Let us first discuss the behavior of the coefficient λ_0 ', which with fixed properties of the components depends only on the concentration of the solid phase and is measured at a state of rest in a zero field. The relation $\lambda_0'(\phi)$ is shown in Fig. 1. It should be recalled that the values of λ_0' are measured in units of the thermal conductivity of the carrier.

Returning to experimental data on the concentration dependence of the thermal conductivity of magnetite magnetic fluids [9], we find the empirical formula $\lambda_0' = 1 + k q_m$ with k = 4.5 for the initial section of this relation. In accordance with Eq. (3.12), we have $\lambda_0' = 1 + k' \varphi$, $k' = 3(\Lambda - 1)/(\Lambda + 2)$ on the linear section. Meanwhile ($\Lambda = 48$) k' = 2.82 for magnetite in kerosene. The difference between k' and k can be attributed to the difference in the concentration of the solid phase φ from the magnetic concentration φ_m used in the empirical relation in [9]. This difference, in turn, exists because of the presence on the particles of a nonmagnetic surface layer [10] with a thickness on the order of the lattice constant $\delta = 8 \cdot 10^{-4} \mu m$. This layer forms with the chemical deposition of a surface-active substance (surfactant). Taking the mean radius of the magnetic core of the particle R to be equal to $5 \cdot 10^{-3} \mu m$, we find $\varphi = \varphi_m (1 + \delta/R)^3 = 1.56 \varphi_m$. From this, $k = k' \varphi/\varphi_m = 4.4$, which agrees with the experimental data to within 2%. The points in Fig. 1 show the data in [9] for magnetite in kerosene recalculated with the use of the above relation between φ and φ_m .

We begin our examination of transport by internal rotation with an analysis of an experiment involving intensification of heat transfer through a magnetic fluid rotating about the axis of a cylindrical layer under the influence of a transverse magnetic field [5]. In this



geometry, $g \cdot v = 0$, and the measured increment of the coefficient of effective thermal conductivity is numerically equal to λ_2' . Since (as noted above) a marked increase in heat transfer in a magnetic fluid due to rotation is possible only in the presence of aggregates, it is necessary to evaluate the thermophysical properties of the latter. The magnetic concentra tion of particles in the test specimen was 0.06, so the concentration of the solid phase was 0.094. The fraction of the volume occupied by the aggregates was equal to the hydrodynamic concentration — including the volume occupied by surfactant layers. The hydrodynamic concentration is usually evaluated from the increment of the effective viscosity of the magnetic fluid in the field. However, such estimates yield very conflicting results, since this increment also depends to a significant extent on the form of the particles (aggregates) - which is usually unknown. We will determine it from the formula $\Psi_{\rm h} = \Psi(1 + \delta_1/R_1)^3$, where δ_1 is the length of a surfactant molecule. It is equal to $2 \cdot 10^{-3}$ µm for oleic acid. The quantity R_1 in the formula is the radius of the particle. Taking R_1 = 5.10^{-3} μm and Ψ = 0.094, we obtain $\varphi_h = 0.25$. In the present case, the volume fraction of particles in aggregates $\varphi' = \varphi/\varphi_h = 0.38$. Using this value, we find from Eq. (3.10) for the thermal conductivity of the aggregate $\lambda_1^0 = 2.7\lambda_2^0$ (λ_2^0 is the thermal conductivity of kerosene), i.e., Λ = 2.7. Calculating the heat capacity of a unit volume of aggregate from the formula c' = $\rho_b \varphi' c_b + \rho_f (1 - \varphi') c_f$, we also obtain an estimate for its diffusivity $\kappa_1 = \lambda_1^0 / c' = 1.6 \cdot 10^{-7}$ m²/sec. Then S = κ_1/κ_2 = 2.1. Figure 2 shows coefficients λ_1' and λ_2' calculated with Λ = 2.7, S = 2.1, and φ = 0.25 and referred to the thermal conductivity of the colloid λ_0' = 1.29. For comparison, we take the data in [5] in a Maxwell field (H = 1500 Oe), when the

aggregates are least subject to hydrodynamic disintegration and are nearly completely stopped by the field. Here they rotate inside the fluid with the empirically measured rate of rotation of the layer. We have a single undetermined parameter — the size of an aggregate a to establish agreement between theory and experiment. Agreement is obtained with a choice of $a = 110 \ \mu\text{m}$. Data from [5] recalculated in accordance with this value is shown by the points in Fig. 2. The agreement between theory and experiment is very good. We should point out the qualitative aspect of the relation: λ_2 ' increases quadratically on the initial section and then slows, with the increase becoming close to linear. As regards the quantitative value of α , it may be overstated due to the nonspherical form of aggregates stretched out by the field. In accordance with Fig. 2, the increase in λ_1 ' with Pe₂ is at first linear and then also slows.

These relations are shown in Fig. 3a, b in a logarithmic scale within a broader range of Pe₂ for $\Psi = 0.25$. The calculations were performed up to Pe₂ $\approx 10^3$. At large values of Pe₂, the series for the functions Φ_2 and F_2 diverge. It should be recalled that such large values of Pe2 exceed the earlier-indicated boundary connected with the use of the Stokes profile for flow in a microscopic eddy. The coefficients λ_1'/λ_0' and λ_2'/λ_0 are greater for particles with a lower thermal conductivity. Meanwhile, we have an approximate equality (in order of magnitude) $\lambda_1'^2 \approx \lambda_2'$. It is interesting that the functions $\lambda_1'(\text{Pe}_2)$ and λ_2 '(Pe₂) become nonmonotonic for large values of Pe₂. This is difficult to explain, but such behavior persists with an increase in the number of terms in the series and doubling of the accuracy of the entire computation. The problem of the divergence of the series for F_2 and Φ_2 also arises for low concentrations. As a result, the concentration dependence of the coefficients λ_1' and λ_2' was calculated only for $\Psi \ge 0.2$. The results are shown in Fig. 4a, b (curves 1, 3, and 4 for Pe₂ = 100, 15, and 10). The calculations were performed up to $\varphi = 1$. However, they lose meaning (as indicated by the dotted lines) as a dense packing is approached. The dashed lines show the results of extrapolation to the region of low concentrations. It can be seen that the character of the relations is to a large extent determined by the relationship between the thermophysical properties of the fluid and particles. Thus, the relation $\lambda_1'/\lambda_0' = f(\varphi)$, increasing monotonically with φ for the impermeable particles, acquires a gentler slope for a suspension with a fluid and particles having roughly the same thermophysical characteristics (aggregated magnetic fluid). The relation exhibits a fairly sharp maximum in the neighborhood $\varphi \simeq 0.25$, when the thermal conductivity of the particles is considerably greater than the thermal conductivity of the carrier fluid. The relation $\lambda_2'/\lambda_0' = f(\varphi)$ is monotonic in character when the thermophysical properties of the particles and fluid are the same; the maximum appears when the thermal conductivity of the particles deviates from that of the fluid in either direction.

LITERATURE CITED

- 1. V. G. Bashtovoi, A. N. Vislovich, and B. É. Kashevskii, "Phenomenon of microconvective heat and mass transfer in fluids with internal rotation," PMTF, No. 3 (1978).
- 2. A. O. Tsebers, "Some features of transport phenomena in suspensions with internal rotation," Prikl. Mat. Mekh., <u>43</u>, No. 4 (1978).
- 3. M. A. Martzenyk and V. I. Thernatinskii, "Transverse heat transport in ferrofluid in rotating magnetic field," IEEE Trans. Magn., <u>MAG-16</u>, No. 2 (1980).
- 4. N. I. Ivanova, "Development of methods of studying heat and momentum transport processes in magnetizing fluids with internal rotation," Author's Abstract of Candidate's Dissertation, Physico-Mathematical Sciences, ITMO AN Belorussian SSR, Minsk (1986).
- 5. B. É. Kashevskii and N. I. Ivanova, "Heat transfer by internal rotation in magnetic fluids," Magn. Gidrodin., No. 3 (1985).
- G. N. Dul'nev and Yu. P. Zarichnyak, Thermal Conductivity of Mixtures and Composites [in Russian], Énergiya, Leningrad (1974).
- 7. D. Joseph, Stability of the Motions of a Fluid [Russian translation], Mir, Moscow (1981).
- 8. F. G. Bar'yakhtar, Yu. I. Gorobets, et al., "Hexagonal grid of cylindrical magnetic domains in films of a ferrofluid," Magn. Gidrodin., No. 3 (1981).
- 9. G. E. Kronkalns, M. M. Maiorov, and V. E. Fertman, "Temperature dependence of the physical properties of magnetic fluids," Magn. Gidrodin., No. 2 (1984).
- R. Kaiser and G. Miskolcty, "Magnetic properties of stable dispersion of subdomain magnetic particles," J. Appl. Phys., <u>41</u>, No. 3 (1970).